PAPER TITLE
Capital Flows, Public Policies and Emigration Patterns from a Developing Country

PAPER AUTHOR
Rajat Acharyya
&
Saibal Kar

WP NO 3
Capital Flows, Public Policies and Emigration Patterns from a Developing Country

Rajat Acharyya, Department of Economics, Jadavpur University, Kolkata, India
*Saibal Kar, Centre for Studies in Social Sciences, Calcutta and IZA, Bonn

January 2013

Abstract: Microeconomic models of migration networks discuss migration patterns of prospective emigrants. We adopt an open economy model to show that stock of emigrants influences future flows even at aggregate levels. We develop a general equilibrium model for a developing country to establish that trade liberalization, stock of skilled and unskilled emigrants, migrant remittances and capital investments critically influence rates of emigration. Interestingly, capital investments may raise emigration prospect among skilled workers at source and lower that for unskilled types. When domestic wage is sensitive to emigration, trade reform raises skill emigration but lowers unskilled emigration. Finally, we establish that investments by skilled emigrants lower unskilled emigration while a restriction on unskilled emigration raises skilled emigration.

Keywords: Trade, emigration, skilled labor, specific factor, remittances, investments
JEL Classification: F22, J64, O15

*Corresponding Address: Centre for Studies in Social Sciences, Calcutta
R-1, B. P. Township, Kolkata 700 094, India
E-mail: saibal@cssscal.org

Acknowledgements: Saibal Kar thanks conference participants at the Western Economic Association Meeting held in Tokyo, 2013 and Sugata Marjit for useful comments and the Centre for Training and Research in Public Finance and Policy (CTRPFP, CSSSC) for financial support.
Capital Flows, Public Policies and Emigration Patterns from a Developing Country

Abstract: Microeconomic models of migration networks discuss migration patterns of prospective emigrants. We adopt an open economy model to show that stock of emigrants influences future flows even at aggregate levels. We develop a general equilibrium model for a developing country to establish that trade liberalization, stock of skilled and unskilled emigrants, migrant remittances and capital investments critically influence rates of emigration. Interestingly, capital investments may raise emigration prospect among skilled workers at source and lower that for unskilled types. When domestic wage is sensitive to emigration, trade reform raises skill emigration but lowers unskilled emigration. Finally, we establish that investments by skilled emigrants lower unskilled emigration while a restriction on unskilled emigration raises skilled emigration.

Keywords: Trade, emigration, skilled labor, specific factor, remittances, investments
JEL Classification: F22, J64, O15
1. Introduction

Migrant remittances significantly enhance welfare among the remaining residents in developing and transition countries. One of the main benefits of inward remittances is relaxing the financial constraint for potential migrants in addition to usual enhancement in consumption of durable and non-durable goods. Mueller and Sharif (2011) use a propensity score matching approach to show that remittances received from internal migrants in India cast positive and significant influence on school attendance of teens. At the same time, it is now documented by several sources that migrants themselves and through creation of migrant networks, source countries receive large inflow of capital meant primarily for business. It also seems that the two types of capital movements arise separately from various skill types: the skilled are more likely to invest directly in the source country, whereas the relatively unskilled are more likely to remit for livelihood support of the non-migrant family. These two sources offer a large inflow of capital, of which remittances alone account for 6-8% of GDP in some of the smaller countries (Appendix 1, figures 4-8).

This paper models the impact of capital investments by skilled workers and inward remittances by relatively unskilled workers on the pattern of emigration among remaining residents, comprising of skilled and unskilled workers. In recent empirical work (Zhao, 2003) it has been verified that stock of experienced migrants influences further flows in China. In our paper, migration decisions are modeled under the assumption that domestic wages are sensitive to emigration, in general. In particular, post emigration, wages respond to both inward remittances and capital investments, but quite differently as we show. Note that, the growing literature on impact of remittances and
capital flows on issues such as emigration decision are often tested with micro-data, but remains deficient of theoretical premises. The present paper is an attempt at filling this gap.

It is well known that various domestic economic policies, poverty, inequality, etc. influence labor migration. The factors may be different for long-run vis-à-vis short-run migration patterns as described in Narayan and Smyth (2006) for migration between Fiji and the US. Further, long run patterns of migration may be influenced by who migrates more. As empirically verified (see Acharyya and Kar, 2005; Kar 2009, for India) a considerably large section of emigrants are unskilled by nature. The unskilled emigrants maintain strong ties with their family in the source country and send inward remittances for consumption support. Compared to this, skilled workers tend to invest (instead of regular remittances) in their home country in view of post-retirement homecoming (viz. Taylor, 2006).\footnote{These statements do not imply that skilled do not send remittances at all and likewise for investments by the unskilled. In fact, for unskilled households part of the remittance receipts are spent on education and durable assets, which may be treated as investments. Notwithstanding, consumption of wage goods out of remittances by relatively unskilled households and pure capital market transactions by relatively skilled emigrants are quite significant (viz. Acharyya and Kar, 2005, describe skill composition of Indian emigrants and remittance to non-remittance transfers).} None of the available papers discuss these two phenomena jointly in order to investigate their impact on emigration prospect among remaining residents.

At the micro/household level, remittance is used for various purposes – smoothing consumption between good states and bad states of nature, financing human capital, accumulation of physical assets, purchasing life and asset insurances, etc. It also helps to relax budget constraint for potential emigrants within the family. Although various implications of remittances have been studied, the role of investment by skilled emigrants received little attention. We show that skilled and unskilled emigration patterns may
strongly but in polar ways interact with each other in the presence of inward remittance
vis-à-vis inward capital flows.

While Beine, Docquier and Ozden (2011, using stock of diaspora abroad), Rapoport and Docquier (2006) and Taylor (2006) provide detailed and elegant compendiums on diasporas and their economic interactions with source and host countries, Calero, Bedi and Sparrow (2009), Vargas-Silva (2008), De Luna-Martinez (2005), Patterson and Reincke (2005) and others offer strong empirical support in favor of migration-remittance links across the developed and developing world. Interestingly, Dimova and Wolff (2009) use a case study for chain migration on Bosnia-Herzegovina where remittances strongly benefit potential emigrants. Moreover, remittance contributes significantly to GNP of many recipient countries and despite country-specific/regional variations aggregate receipts have risen steadily over time (figures 4-8, Appendix I).

Despite these contributions, analysis of the impact of emigration stock on emigration patterns among skilled and unskilled workers is absent in the current literature. There is also a general neglect of the complex roles domestic production and factor reallocation plays in influencing emigration decisions. Our model brings together these features. We observe that the stock of emigrants by their actions and the extant trade policies in the source country directly influence flow of prospective emigrants.

Section 2 develops a model where emigration affects equilibrium domestic wage and we explore decision patterns among prospective emigrants. We prove that when wage is sensitive to emigration, trade liberalization reduces skill emigration but raises outflow of unskilled workers. Section 3 summarizes these findings and concludes.
2. **Decision to Emigrate**

This section argues that in many developing and transition countries, far less populous than China and India, domestic wages may change due to emigration.\(^2\) The equilibrium domestic wages are sensitive to emigration, which in turn depends on the direct and indirect costs of emigration. We use emigration cost to account for the network effect within each skill type. Assume that the stock of emigrants of a particular skill type lowers the cost for prospective emigrants of the same/other types directly. Remittance at the household level is one channel through which this network functions. But, even beyond family ties, emigrant stock may help to lower information gaps (and uncertainties, viz. Katz and Stark, 1987; Dequiedt and Zenou, 2011) for prospective emigrants of similar skill types and hence the cost.\(^3\)

Before coming to the cost equations, let us assume that three final goods are produced in a developing country and sold at world prices, \((P_j^*, j = X, Y, Z)\).

\[
X = X(S_X, K_X) \quad \text{is an export good manufactured with the help of skilled workers (S) and capital (K).}
\]

\[
Y = Y(L_Y, K_Y) \quad \text{is a tariff-protected \((t_Y)\) import competing good using relatively unskilled labor (L) and capital.}
\]

\[
Z = Z(L_Z, T_Z) \quad \text{is an agricultural non-traded good using land (T) and unskilled labor (L) as factor inputs.}
\]

Note that, the choice of product types helps to construe a typical developing country structure, where large

---

\(^2\) Recalling the Romanian evidence (Wesselingh, 2010), the government has revised Doctor’s pay from Euro 220 to Euro 400 in the wake of emigration wave. This may slow down exodus of large numbers of anesthetists, cardiologists, urologists and neurosurgeons compared to the Polish experience.

\(^3\) Katz and Stark (1987) use a discount rate on foreign income defined as the psychological/cultural costs of emigration from poor to rich countries. We assume that existing stocks lowers such cost for prospective emigrants of same skill types by cushioning cultural/ethnic distances created by emigration or by directly sending in remittances. Dequiedt and Zenou (2011) also calculate income earned by individuals in both source and destination countries under regimes of information asymmetry.
sections of relatively unskilled workers are engaged in traditional manufacturing and agriculture. The determination of factor prices involves all three sectors (Appendix II). Further extensions of this model with relation to issues in land redistribution, agricultural prospects, etc. should enrich the existing set of results and currently shelved for future attempts. The production functions are homogeneous of degree one in inputs. Product and factor markets are competitive. The unskilled labor is freely mobile between agriculture and industry.

The following set of equations describes this economy. Expressions (1-3) are competitive price equations where unit cost equals unit prices of each commodity on the right hand side. Equations (4-7) are full-employment conditions. Rate (or percentage) of emigration is endogenous for skilled and unskilled types and denoted by \((\alpha, \beta)\), respectively. We observe how two different forms of capital movements and trade reform affect \((\alpha, \beta)\) vis-à-vis the steady state equilibrium. Changes in \((\alpha, \beta)\) capture changes in the rate of emigration.

\[
\begin{align*}
\text{(1)} & \quad w_a a_{sx} + r a_{kx} = P_x \\
\text{(2)} & \quad w a_{ly} + r a_{ky} = P_y (1 + t_y) \\
\text{(3)} & \quad w a_{lz} + R a_{tz} = P_z \\
\text{(4)} & \quad a_{sx} X = S(1 - \alpha) \\
\text{(5)} & \quad a_{ly} Y + a_{lz} Z = L(1 - \beta) \\
\text{(6)} & \quad a_{tz} Z = \bar{T} \\
\text{(7)} & \quad a_{kx} X + a_{kz} Y = \bar{K}
\end{align*}
\]

As discussed earlier, there are both direct and indirect costs of emigration. Usually, if the
domestic wage is lower than the foreign wage less the cost of emigration for each skill type, migration occurs. We assume that \( \delta_s(\alpha) \) and \( \delta(\beta) \) are costs of emigration for skilled and unskilled types respectively. Equations (8) and (9) offer migration equilibrium conditions for the two types, where the RHS denote net foreign wages earned less the cost of emigration for each type as function of its own stock.

\[
\begin{align*}
W_s &= W_s^* - \delta_s(\alpha) \\
W &= W^* - \delta(\beta)
\end{align*}
\]

Equations (1)-(9) contain nine variables that are fully determined by these equations.\(^4\) If cost of migration falls with more of a given skill type living abroad already, then \( \delta_s'(\alpha) < 0 \) and \( \delta'(\beta) < 0 \). However, the cross effects across skill types are assumed negligible. This means that the cost of emigration for unskilled types is not directly affected by stock of skill emigrants and vice versa. Note that, from (8) and (9) it is immediate that, increase in the number of emigrants raises the home country wages. This is obvious as emigration lowers the supply of workers at home. Thus, both skilled and unskilled wages increase in the number of skilled and unskilled emigrants. On the other hand, given the foreign skilled (unskilled) wage, as more skilled (unskilled) workers emigrate, the cost of emigration declines and therefore the net foreign skilled wage rises. However, we continue to assume that the immigrant flow is not large enough to affect foreign wages. Hence, both \( W_s^* - \delta_s(\alpha) \) and \( W^* - \delta(\beta) \) asymptotically approaches \( W_s^* \) and \( W^* \) respectively. For any given \( \beta \), the skilled emigration is \( \alpha(\beta) \) for which home skilled wage equals the net foreign skilled wage.

---

\(^4\) Equations (1-9) is a system with specific factors. It solves for equilibrium values of nine variables using the same mechanism as in Jones (1971).
Given the steady state equilibrium, let us consider a thought experiment. An inflow of remittances to source country households takes place. It raises income but as commodity prices are frozen from outside, it does not affect factor prices and hence rates of migration do not change. But there are other channels, which may affect \((\alpha, \beta)\). For example, \((A)\) tariff is reduced for sector \(Y\); \((B)\) inward remittance/capital flows in following trade reform; and \((C)\) unskilled workers face restrictions to emigrate.

Standard calculations, as shown in Appendix II lead to changes in the two wages that hold the key here. These are expressed in terms of changes in emigration rates and the level of tariff:

\[
\hat{w}_s = \frac{\mu}{A} \lambda_{XX} \hat{\alpha} + \frac{\rho}{A} \lambda_{LY} \hat{\beta} - \frac{B}{A} \hat{I}_y \tag{10}
\]

\[
\hat{w} = \left[ \frac{1}{\theta_{LY}} - \frac{\theta_{KY}}{\theta_{LY}} \frac{\theta_{SX}}{\theta_{KX}} \frac{B}{A} \right] \hat{\alpha} + \frac{\mu}{A} \lambda_{SX} \frac{\theta_{KY}}{\theta_{LY}} \frac{\theta_{SX}}{\theta_{KX}} \hat{\alpha} + \frac{\rho}{A} \lambda_{LY} \frac{\theta_{KY}}{\theta_{LY}} \frac{\theta_{SX}}{\theta_{KX}} \hat{\beta} \tag{11}
\]

where, \(\gamma = \frac{t_y}{1 + I_y}\), \(\mu = \frac{\alpha}{1 - \alpha}\), \(\rho = \frac{\beta}{1 - \beta}\), \(B = \frac{\lambda_{KY}}{\lambda_{LY} \lambda_{LX}}\), and

\[
A = \left[ \sigma_x \frac{\lambda_{XX}}{\theta_{XX}} + \sigma_y \frac{\lambda_{KY}}{\theta_{KX}} \frac{\theta_{SX}}{\theta_{LY}} + \sigma_z \frac{\lambda_{KZ}}{\theta_{LZ}} \frac{\theta_{KX}}{\theta_{LY}} \frac{\theta_{SX}}{\theta_{KX}} \right]
\]

The migration equations (8) and (9) indicate that local wages and number of workers emigrating in equilibrium must be simultaneously determined. Figures 1 and 2 illustrate simultaneous determination of these variables. Figure 1a depicts the emigration equilibrium for the skilled workers, given the number of unskilled emigrants \((\beta)\) and the

---

5 Section 3 discusses results when small country assumption is relaxed.
6 Countries like China and India have emigration policies restricting emigration of unskilled/skilled workers. The Ministry of External Affairs, Govt. of India practices ECNR (Emigration Check not Required) criterion to monitor exodus of unskilled workers. Only the skilled (high school graduate and above) are exempt from emigration check by concerned authorities. However, restriction may also come from the host country, like point-based immigration criteria limiting inflow of unskilled/skilled workers.
tariff rate \((t_Y)\). Figure 1b, on the other hand, depicts the same for unskilled workers. In Figure 1b, the interpretation of \(\beta(\alpha)\) holds similarly. From these two figures, it is easy to check that an inverse relationship holds between emigration of skilled and unskilled workers. For a larger stock of unskilled emigrants \((\bar{\beta} > \beta)\), less skilled workers will emigrate and vice versa. These optimum emigration decisions are shown in Figure 2 with equilibrium emigration levels being \(\alpha^*\) and \(\beta^*\).

![Migration Decision Diagram](image)

**Figure 1: Migration Decision**
Result 1: If emigration is sensitive to domestic wages, a tariff reduction in the import competing sector lowers skilled emigration but raises unskilled emigration.

Proof: Let us consider three comparative static exercises as follows.

(A) The effect of a reduction in the tariff rate can be easily traced out from Figures (1) and (2). Using (10) and (11), a reduction in tariff rate raises $w_s$ and lowers $w$, since

\[ \left[ \frac{1}{\theta_{LY}} - \frac{\theta_{KY}}{\theta_{LY}} \frac{\theta_{SX}}{\theta_{KX}} \frac{B}{A} \right] > 0. \]

Hence, a tariff reduction lowers the incentive for migration among the skilled while raising it among the unskilled workers. This asymmetric impact on emigration pattern changes wages further. This must create newer incentives and disincentives for emigration and this cycle continues until the final changes in the emigration levels settle in. The final change in emigration is a magnification of the initial...

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure2.png}
\caption{Equilibrium Emigration Levels}
\end{figure}
decline in emigration of skilled workers and rise in the emigration of unskilled workers (as shown in Figure 2). Intuitively, if trade liberalization in the country allows skill biased technological change and hence improvement of skilled wage, skilled workers may be less inclined to emigrate. Conversely, if the unskilled workers suffer vis-à-vis skilled workers in the labor market, the rate of emigration may increase in the post liberalization period. Result 1 summarizes these.

Here, as local wages change with emigration a tariff reduction unambiguously lowers emigration of the skilled workers. A reversal of the effect on skilled emigrants is due to the fact that tariff reduction raises skilled wage at home and thus works as a disincentive towards emigration. This also highlights the role of push factor relative to pull factor in the migration decision. The other two comparative static exercises are discussed in section 2.1.

2.1 Investments and Remittances

Consider capital inflow and remittance as potential sources of change in the emigration pattern. First, referring to the capital constraint in (7) and taking percentage changes along with a small increment in the stock of capital (due to investments by skilled emigrants):

\[
\lambda_{kx} (\hat{X} + \hat{a}_{kx}) + \lambda_{ky} (\hat{Y} + \hat{a}_{ky}) = \hat{K}
\]

(12)

Proceeding as before,

\[
\hat{w}_s = \left[ \frac{\mu}{A} \frac{\lambda_{kx}}{\lambda_{sx}} \hat{\alpha} + \frac{\rho}{A} \frac{\lambda_{ky}}{\lambda_{ly}} \hat{\beta} - \frac{B}{A} \hat{\gamma}_y + \frac{1}{A} \hat{K} \right]
\]

Similarly,
\[
\hat{w} = \left[ \frac{1}{\theta_{LY}} - \frac{\theta_{KY}}{\theta_{LY}} \frac{\theta_{SX}}{\theta_{KX}} \right] \hat{y} + \frac{\mu \lambda_{KX} \theta_{KY} \theta_{SX} \theta_{KX}}{A \lambda_{LY} \theta_{LY} \theta_{KX}} + \frac{\rho \lambda_{KY} \theta_{KY} \theta_{SX}}{A \lambda_{LY} \theta_{LY} \theta_{KX}} \frac{1}{\theta_{LY} \theta_{KX}} \hat{K}
\]

So, large investments by skilled emigrants raises both \( w_s \) and \( w \). For any given \( \beta \), \( w_s \) rises while lowering \( \alpha \). On the other hand, given \( \alpha \), \( w \) rises and it lowers \( \beta \) in turn.

At the new equilibrium, \( \beta \) unambiguously falls. But \( \alpha \) may fall or rise (Figure 3). While the rate of emigration for unskilled workers fall, it is quite possible that the same does not hold true for skilled workers. They may emigrate more than before generating an asymmetric effect across skill types. In other words, investment by skilled emigrants lowers the incentives for unskilled workers to emigrate but such investment may not be a sufficient pre-condition for lowering incentive for emigration among the skilled workers.

Note that, a fall in \( \beta \) lowers \( w_s \) and subsequently, there will be an induced rise in \( \alpha \).

![Figure 3. Investment and Emigration](image-url)
As for inward remittances, we have already discussed that unlike the increment to capital stock, it does not affect domestic output as commodity prices are given from outside. Therefore without changes in $w$, $w_s$, $r$ and $R$, $\beta$ and $\alpha(\beta)$ do not undergo any change. Thus, remittance does not affect migration decision in this case.

**Result 2:** Investment/remittances by skilled emigrants unambiguously lowers unskilled emigration but may raise skilled emigration. On the other hand, emigration restriction on unskilled workers leads to greater emigration of skilled workers.

*Proof:* The following comparative static exercise is used as the proof.

(C) Physical restrictions on immigration of unskilled workers by the host country to say, $\bar{\beta} < \beta^*$ will induce more skilled workers to emigrate from the source country (see, Dixon, Rimmer and Roberts, 2013 for US policy on restricting illegal immigrants). The intuition is that, restriction on emigration would lower $w$ in the source country from the level that is attained when emigration up to $\beta^*$ is free. In other words, $w(\bar{\beta}) < w(\alpha^*, \beta^*)$. This further implies that, a lower $w$ in the event of restricted emigration raises $r$ in equilibrium and lowers $w_s$. Hence, the gap between skilled wage at home vis-à-vis the foreign country $[w_s^* - \delta_S(\alpha) - w_s]$ widens and more skilled workers emigrate. This leads to Result 2 above.
3. Discussion and Concluding Remarks

The welfare implications of emigration from a source country on the remaining residents have received much attention in the literature. The micro-evidence from a wide range of countries in Asia, Africa, Latin America, the Caribbean etc. show that remittances in cash or kind directly mould the consumption pattern of remaining residents in households where one or more members has emigrant status. Yet, the connection between remittances, capital investment and migration prospect is somewhat unclear and demands theoretical support.

We have been able to develop some connections along this line. The impact of inward remittances and/or large foreign capital inflows appear to be asymmetric across skilled and unskilled residents as far as it captures prospects of emigration. We established that there is strong synergy between migration decisions undertaken by each type when countries undergo trade liberalization, receive capital or face emigration restriction. Before we conclude, let us deliberate briefly on the role of remittances in particular.

With respect to remittance inflow to relatively unskilled households, section 2 has already discussed the possibility generated by the stylized model. However, one should acknowledge that results may differ when the small country assumptions is dropped. When this model is extended to large countries, remittance works in the following way. It raises income and if all goods are normal goods, domestic demand for all goods must rise. This means, exports will fall and import demand will rise at initial set of world prices. Hence, if the host country is large, all world prices should rise. But these price
changes need not be proportional and relative magnitudes should depend on two things: (a) income elasticity of domestic demand and (b) foreign price elasticity of demand and supply. For example, if all goods are unitary income elastic in the host country, remittances will raise their demand equally. Hence, we can expect decline in host country export supply and increase in host country import demand to be of same magnitude. Yet, depending on the slope (and price elasticity) of world demand curve for what the host country exports and of world supply curve for what it imports, world price changes may be non-proportional. This should have further implications for factor prices and hence migration decisions. However, given that larger /richer countries receive negligible remittance transfers as compared to their GDP, income effects are also negligible.

With relation to the so far neglected agricultural sector, suppose there is a technical improvement in land use for agricultural production. It should raise \( Z \), and to support that labor must be drawn from sector \( Y \), a pattern of reverse internal migration that is fairly common in most developing countries. This raises the unskilled wage, lowers return to capital and subsequently raise skilled wage. Not surprisingly, one then expects rate of both skilled and unskilled emigration to fall from the migration equilibrium in (8) and (9).

Finally, it was also shown that when domestic wage is sensitive to emigration, a fall in tariff protection in the import sector lowers skill emigration but raises unskilled emigration. However, if the skilled emigrants invest in their home country it raises output of the import competing good and hence, emigration for remaining residents (unskilled types) falls. As an interesting policy-relevant support to this result, we showed
that if the source country imposes emigration restriction on unskilled workers, it lowers domestic unskilled wage, raises return to capital and lowers skilled wage. This contributes to greater exodus of skilled workers from the country. Without a general equilibrium modeling of such complex interplays, many policies designed for emigrants may fail to attain desired outcomes.
References


Adams Jr., Richard (2009), The Determinants of International Remittances in Developing Countries, World Development, 37, 1, 93-103.


Calero, C, A. Bedi and R. Sparrow (2009), Remittances, Liquidity Constraints and Human Capital Investments in Ecuador, World Development, 37, 6, 1143-1154


Appendix 1

Figure 4. Graph Showing Number of Migrants from Select Source Countries

![Graph Showing Number of Migrants from Select Source Countries](image1)

Source: World Development Indicator, various years, World Bank and own calculations

Figure 5. Aggregate Workers’ Remittance by Broad Country-Types

![Aggregate Workers’ Remittance by Broad Country-Types](image2)

Source: World Development Indicator, various years, World Bank and own calculations
Figure 6. Aggregate Workers’ Remittance by Regions in the World

Source: World Development Indicator various years, World Bank and own calculations

Figure 7. Remittances as a Share of GDP

Source: World Development Indicator, various years, World Bank and own calculations
Figure 8. Share of Remittances Received by most Important Recipients

Source: World Development Indicator, World Bank and own calculations.

Appendix II

Changes in Local Wages

From the full employment of labor condition (5) in the text, the following proportional change can be obtained:

$$\lambda_{LY} (\hat{Y} + \hat{a}_{LY}) + \lambda_{LZ} (\hat{Z} + \hat{a}_{LZ}) = -\rho \hat{\beta} \quad (A.1)$$

where, $\rho = \frac{\beta}{1 - \beta}$.

Similarly, from the full employment conditions for land (6) we get: $\hat{Z} = -\hat{a}_{TZ}$.

Substitution of this in (A.1) yields,

$$\lambda_{LY} (\hat{Y} + \hat{a}_{LY}) + \lambda_{LZ} (\hat{a}_{LZ} - \hat{a}_{TZ}) = -\rho \hat{\beta}$$
Using the definition of factor substitution elasticity in \( Z \), \( \sigma_\text{Z} = \frac{\hat{a}_{\text{LZ}} - \hat{a}_{\text{TZ}}}{\hat{R} - \hat{w}} \), and the relationship between change in return to land and the unskilled wage at zero profit condition, \( \hat{R} = -\frac{\theta_{\text{LZ}}}{\theta_{\text{TZ}}} \hat{w} \). This boils down to:

\[
\hat{Y} = \hat{a}_{\text{LY}} + \sigma_\text{Z} \frac{\lambda_{\text{LZ}}}{\lambda_{\text{LY}} \theta_{\text{TZ}}} \hat{w} - \frac{\rho}{\lambda_{\text{LY}}} \hat{\beta} \quad (A.2)
\]

On the other hand, from the full employment condition for skilled workers we get,

\[
\hat{X} = -\hat{a}_{\text{SX}} - \mu \hat{\alpha} \quad (A.3)
\]

Finally, the full employment condition for capital implies,

\[
\lambda_{\text{KK}} (\hat{X} + \hat{\alpha}_{\text{KK}}) + \lambda_{\text{KY}} (\hat{Y} + \hat{\alpha}_{\text{KY}}) = 0 \quad (A.4)
\]

Substitution of (A.2) and (A.3) in (A.4) yields:

\[
\lambda_{\text{KK}} (\hat{a}_{\text{KK}} - \hat{a}_{\text{SX}}) - \mu \lambda_{\text{KK}} \hat{\alpha} + \lambda_{\text{KY}} (\hat{a}_{\text{KY}} - \hat{a}_{\text{LY}}) + \sigma_\text{Z} \frac{\lambda_{\text{KY}} \lambda_{\text{LZ}}}{\lambda_{\text{LY}} \theta_{\text{TZ}}} \hat{w} - \rho \frac{\lambda_{\text{KY}}}{\lambda_{\text{LY}}} \hat{\beta} = 0 \quad (A.5)
\]

Using factor substitution elasticities in production of \( X \) and \( Y \), \( \sigma_X = \frac{\hat{a}_{\text{KK}} - \hat{a}_{\text{SX}}}{\hat{w}_S - \hat{r}} \) and

\[
\sigma_Y = \frac{\hat{a}_{\text{KY}} - \hat{a}_{\text{LY}}}{\hat{w} - \hat{r}} \quad \text{respectively, } \hat{w} = \frac{1}{\theta_{\text{LY}}} \hat{\mu} \gamma - \frac{\theta_{\text{KY}}}{\theta_{\text{LY}}} \hat{r} \quad \text{and} \quad r = -\frac{\theta_{\text{SX}}}{\theta_{\text{KK}}} \hat{w}_S \quad \text{from the zero profit conditions in the two sector, condition (A.5) boils down to the expression for the change in skilled wage as specified in (10) in the text and reproduced below:}

\[
\hat{w}_S = \frac{\mu}{A} \lambda_{\text{KK}} \hat{\alpha} + \frac{\rho}{A} \lambda_{\text{KY}} \hat{\beta} - \frac{B}{A} \hat{\mu} \gamma
\]

Similarly, change in unskilled wage, \( \hat{w} = \frac{1}{\theta_{\text{LY}}} \hat{\mu} \gamma - \frac{\theta_{\text{KY}}}{\theta_{\text{LY}}} \frac{\theta_{\text{SX}}}{\theta_{\text{KK}}} \hat{w}_S \) leads to equation (11) above. QED.